



# Methods of Evaluating the Efficiency and Vibration Stability of Vehicles with Internal Combustion Engine

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## Abstract

The internal combustion engine is a source of disturbing oscillations, which affects the energy losses in the transmissions of vehicles. In the paper, using the previously obtained analytical expression for the cyclic elastic coefficient of efficiency of the transmission, the conditions for the appearance of its vibration instability are

determined. An interrelation between the number of ICE cylinders and the elastic and inertial parameters of the transmission taking into account vibration stability has been obtained. As an example, the solution of the problem of replacing a 3-cylinder ICE with a 4-cylinder and a 6-cylinder by an 8-cylinder engine in the design of machine assemblies is considered.

## Analysis of the Latest Achievements in the Publication

A significant number of well-known publications are devoted to the dynamics and methods of studying machine assemblies. These include the classic works of I. I. Artobolevsky, V. L. Weitz, and M. S. Komarov [1, 2, 3, 4]. In the work [3], a classification of the kinematic chains links of a machine unit is presented. The use of internal combustion engines in vehicles that generate fluctuations in the transmission due to the form of torque unevenness leads to the appearance of additional energy losses during the transmission of power to the output shaft. The presence of elastic, inertial, and dissipative links in the transmission leads to circulation and energy dissipation [5, 6]. The influence of these processes on the efficiency of cars and tractors was considered in [7, 8, 9, 10].

In the work [6], an analytical expression was obtained for determining the cyclic elastic transmission efficiency

$$\left(\eta_{tr}^{el}\right)_{cycle} = 1 - \frac{A_M \left(1 - \frac{A_M}{2\bar{M}_i}\right)}{\pi I_{\Pi p} \bar{\omega}_e \omega_M \left(\frac{k^2}{\omega_M^2} - 1\right)} \quad (1)$$

where  $A_M$  - the amplitude of the oscillations of the indicator torque of the engine [6],

$$A_M = 0,5K_1\bar{M}_i, \quad (2)$$

$\bar{M}_i$  - average indicator engine torque;  
 $K_1$  - coefficient of uneven torque [6]

$$K_1 = 0,08 + 14,44 / i_c; \quad (3)$$

$\bar{\omega}_e$  - the average angular velocity of the crankshaft of ICE;  
 $\omega_M$  - the circular frequency of torque fluctuations of engine,

$$\omega_M = 0,5\bar{\omega}_e i_c; \quad (4)$$

$I_c$  - the number of engine cylinders;  
 $k$  - the circular frequency of natural vibrations of the input shaft of the transmission,

$$k = \sqrt{\frac{c_{\Pi p}}{I_{\Pi p}}}; \quad (5)$$

$c_{\Pi p}$  - stiffness of the oscillating system "ICE - transmission - wheeled vehicle - road" brought to the input shaft;

$I_{\Pi p}$  - moment of inertia of the oscillating system "ICE - transmission - wheeled vehicle - road" brought to the input shaft.

It was determined in [10] (see equation (1)) that, at  $A_M = 2\bar{M}_i$ , the value  $\left(\eta_{tr}^{el}\right)_{cycle} = 1$  for any combination of parameters of a motor-transmission installation. The author of [6] determined that this option is implemented when the number of cylinders is  $i_c = 3,68$ . This equality is conditional. The closest integer values to the remarkable point are  $i_c = 3$  and  $i_c = 4$ . But in the same work [6], it was shown that, at  $i_c < 3,68$ , the vibrational processes of transmission should occur in the resonance zone ( $\omega_M > k$ ), and at  $i_c > 3,68$  - it should occur in the pre-resonance zone ( $\omega_M < k$ ).

However, in the work [6], the conditions for ensuring the vibration stability of the engine-transmission installation were not defined and recommendations were not given on the

rational choice of the frequency of free (natural) transmission vibrations for a given number of ICE cylinders.

## The Purpose and Tasks of the Study

The purpose of the study is to ensure a high level of energy efficiency of vehicles with ICE by increasing efficiency due to the rational choice of the frequency of free oscillations of the transmission.

In order to achieve this purpose, it is necessary to solve the following tasks:

- to determine the limit of vibration resistance of the engine-transmission unit;
- to propose methods for rational choice of transmission parameters.

## Statement of Matter Subjects

Equation (1) taking into account equations (2) - (5) takes the following form

$$(\eta_{tr}^{el})_{cycle} = 1 - \frac{\overline{M}_e}{I_{\Pi P}} \frac{(0.006i_c + 1.126) \left(1 - \frac{3.68}{i_c}\right)}{k^2 - \omega_M^2} \quad (6)$$

With the steady state of operation of the engine-transmission units, the average indicator torque of the ICE can be defined as

$$\overline{M}_i = \overline{M}_e / (\eta_{eng})_{cycle} \quad (7)$$

where  $\overline{M}_e$  - the average effective torque of the ICE;  
 $(\eta_{eng})_{cycle}$  - cyclic mechanical engine efficiency.

Equation (6), taking into account (7), takes the form

$$(\eta_{tr}^{el})_{cycle} = 1 - \frac{\overline{M}_e}{I_{\Pi P} (\eta_{eng})_{cycle}} \frac{(0.006i_c + 1.126) \left(1 - \frac{3.68}{i_c}\right)}{k^2 - \omega_M^2} \quad (8)$$

Vibration resistance of the engine-transmission unit will be ensured when the cyclic elastic efficiency is within

$$0 < (\eta_{tr}^{el})_{cycle} \leq 1. \quad (9)$$

From the condition  $(\eta_{tr}^{el})_{cycle} > 0$  we define

$$k^2 - \omega_M^2 > \frac{\overline{M}_e}{I_{\Pi P} (\eta_{eng})_{cycle}} (0.006i_c + 1.126) \left(1 - \frac{3.68}{i_c}\right) \quad (10)$$

From the condition  $(\eta_{tr}^{el})_{cycle} \leq 1$  we obtain the inequality

$$\frac{\overline{M}_e}{I_{\Pi P} (\eta_{eng})_{cycle}} \frac{(0.006i_c + 1.126) \left(1 - \frac{3.68}{i_c}\right)}{k^2 - \omega_M^2} \geq 0 \quad (11)$$

Inequality (11) is satisfied in two cases

$$\begin{cases} 1 - 3.68 / i_c > 0; \\ k - \omega_M > 0, \end{cases} \quad (12)$$

and

$$\begin{cases} 1 - 3.68 / i_c < 0; \\ k - \omega_M < 0. \end{cases} \quad (13)$$

Analyzing the system of inequalities (12) and (13), we can draw the following conclusions:

- at  $i_c > 3.68$ , the engine-transmission unit should be designed so that  $\omega_M < k$  (i.e., at  $i_c \geq 4$ );
- at  $i_c < 3.68$ , the engine-transmission unit should have  $\omega_M > k$  (i.e., at  $i_c \leq 3$ ).

It can be seen from equation (8) that for  $i_c = 3.68$ , the value  $(\eta_{tr}^{el})_{cycle} = 1$  for any ratio of the parameters  $\omega_M$  and  $k$ , and also there is no resonance phenomenon. However, it is clear that the value of  $i_c = 3.68$  is conditional, which cannot be realized. The closest real values are  $i_c = 3$  and  $i_c = 4$ .

We transform equation (11) to the form

$$\frac{I_{\Pi P} (\eta_{eng})_{cycle}}{\overline{M}_e} (k^2 - \omega_M^2) \geq (0.006i_c + 1.126) \left(1 - \frac{3.68}{i_c}\right) \quad (14)$$

Using the similarity theory (the theory of "generalized variable") [12], we introduce a generalized parameter

$$\chi = \frac{I_{\Pi P} (\eta_{eng})_{cycle}}{\overline{M}_e} |k^2 - \omega_M^2| \quad (15)$$

Then equation (14) takes the form

$$\chi > (0.006i_c + 1.126) \left|1 - \frac{3.68}{i_c}\right|. \quad (16)$$

The right-hand side of equation (16) is the minimum allowable value  $\chi = \chi_{\min}$ , which, in turn, is a criterion for the vibration stability of a motor-transmission installation. Thus

$$\chi_{\min} = (0.006i_c + 1.126) \left|1 - \frac{3.68}{i_c}\right|. \quad (17)$$

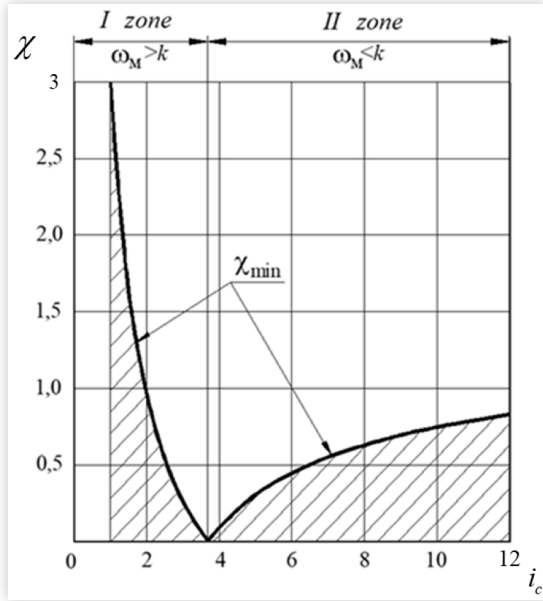
In the Figure 1 shown a graph of the dependence  $\chi_{\min}(i_c)$  and shown the vibration stability zone of engine-transmission units with ICE.

Equation (8), taking into account equations (15) and (17), will take the form

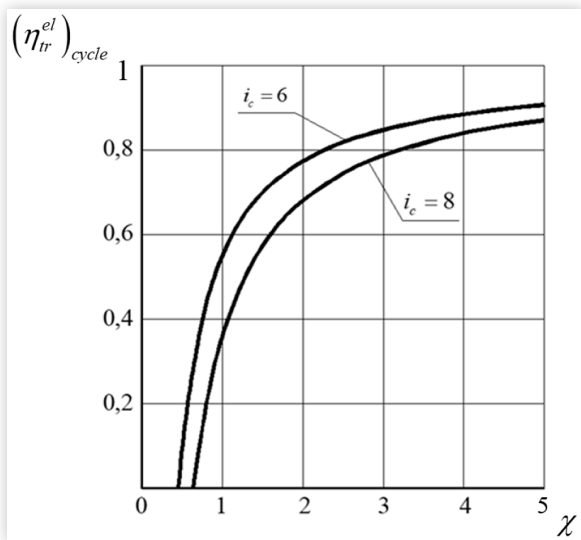
$$(\eta_{tr}^{el})_{cycle} = 1 - \frac{\chi_{\min}}{\chi}. \quad (18)$$

In the figure 2 are shown the dependences of the cyclic elastic efficiency of transmission on the  $\chi$  parameter for 6 and 8-cylinder ICE. Analysis of graphs in figure 2 shows that the

**FIGURE 1** The instability zone of the engine-transmission system with ICE (shaded): I - resonance zone; II - pre-resonance zone



**FIGURE 2** Dependence  $(\eta_{tr}^{el})_{cycle}$  on  $\chi$  for  $i_c = 6$  to  $i_c = 8$



indicated indicator for a 6-cylinder engine is higher than for an 8-cylinder engine (*ceteris paribus*, equal values of the parameter  $\chi$ ). Relative reduction in cyclic elastic transmission efficiency

$$\delta_{\eta} = \frac{(\eta_{tr}^{el})'_{cycle} - (\eta_{tr}^{el})''_{cycle}}{(\eta_{tr}^{el})'_{cycle}}, \quad (19)$$

where  $(\eta_{tr}^{el})'_{cycle}$ ;  $(\eta_{tr}^{el})''_{cycle}$  - are cyclic elastic transmission efficiencies at  $i_c=6$  and  $i_c=8$ .

In the **Figure 3** shown a graph of the  $\chi$  dependence of the cyclic elastic efficiency of the transmission during the transition from a 6-cylinder to an 8-cylinder ICE.

The indicated decrease is 33.6% for  $\chi = 1$  and 4.1% for  $\chi = 5$ . Thus, when installing an 8-cylinder ICE instead of a 6-cylinder ICE, it is necessary to take measures to increase the  $\chi$  parameter from the condition of increasing the cyclic elastic transmission efficiency to the level of the specified efficiency with a 6-cylinder ICE. From the condition

$$(\eta_{tr}^{el})'_{cycle} = (\eta_{tr}^{el})''_{cycle} \quad (20)$$

we find

$$\chi'' = \frac{\chi'_{min}}{\chi''_{min}} \chi' = \frac{0.08}{0.06} \chi' = 1.33 \chi', \quad (21)$$

where  $\chi'$ ;  $\chi''$  - are the values of the generalized parameters with  $i_c = 6$  to  $i_c = 8$ .

The dependency graph (21) is shown in **Figure 4**. An analysis of dependence (21) shows that, when switching from a 6-cylinder to an 8-cylinder internal combustion engine, it is necessary to increase the generalized parameter  $\chi$  by 33%, which will allow maintaining the value of the cyclic elastic transmission efficiency at the same level.

Taking into account that

$$\chi' = \frac{I_{\Pi P} (\eta_{eng})_{cycle}}{M_e''} |k^2 - (\omega_M')^2| \quad (22)$$

and

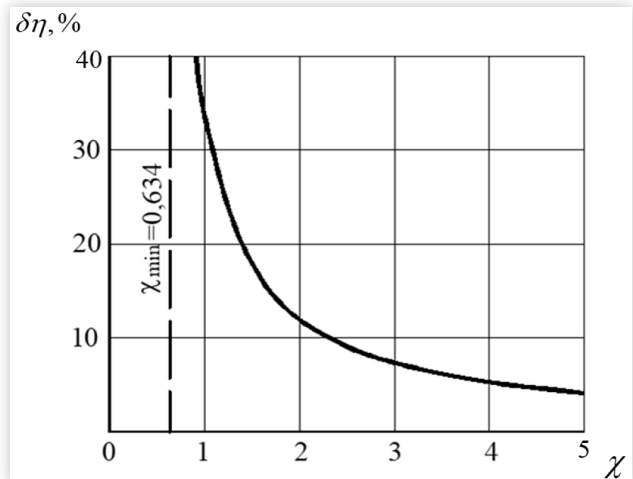
$$\chi'' = \frac{I_{\Pi P} (\eta_{eng})_{cycle}}{M_e''} |k^2 - (\omega_M'')^2| \quad (23)$$

we find

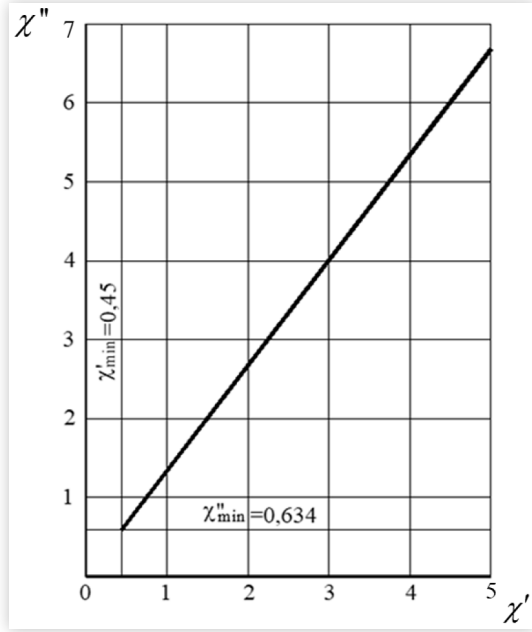
$$\frac{\chi'}{\chi''} = \frac{\overline{M_e}'}{\overline{M_e}''} \frac{k^2 - (\omega_M'')^2}{k^2 - (\omega_M')^2} = 1.33 \quad (24)$$

From **equation (24)** we define

**FIGURE 3** Decrease  $(\eta_{tr}^{el})_{cycle}$  in the transition from  $i_c = 6$  to  $i_c = 8$



**FIGURE 4** The dependence  $\chi'' = f(\chi')$  in the transition from  $i_c = 6$  to  $i_c = 8$



$$\frac{\omega_M''}{\omega_M'} = \sqrt{\frac{k^2}{(\omega_M')^2} \left( 1 - 1.33 \frac{\overline{M}_e''}{\overline{M}_e'} \right) + 1.33 \frac{\overline{M}_e''}{\overline{M}_e'}}. \quad (25)$$

The circular frequency of forced oscillations of the torque can be found from [equation \(4\)](#) and is:

$$\omega_M' = 3\overline{\omega}_e' - \text{with } i_c = 6; \quad (26)$$

$$\omega_M'' = 4\overline{\omega}_e'' - \text{with } i_c = 8. \quad (27)$$

Taking into account [equations \(26\)](#) and [\(27\)](#), we transform [equation \(25\)](#) to the form

$$\frac{\overline{\omega}_M''}{\overline{\omega}_M'} = 0.25 \sqrt{12 \frac{\overline{M}_e''}{\overline{M}_e'} + \frac{k^2}{(\overline{\omega}_e')^2} \left( 1 - 1.33 \frac{\overline{M}_e''}{\overline{M}_e'} \right)}. \quad (28)$$

[Equation \(28\)](#) has a real solution for a non-negative value of the radical expression, i.e. under the condition

$$12 \frac{\overline{M}_e''}{\overline{M}_e'} + \frac{k^2}{(\overline{\omega}_e')^2} \left( 1 - 1.33 \frac{\overline{M}_e''}{\overline{M}_e'} \right) \geq 0. \quad (29)$$

From [equation \(29\)](#) we define

$$\frac{k}{\overline{\omega}_e'} \leq \sqrt{\frac{12}{1.33 - \overline{M}_e'' / \overline{M}_e'}} \quad (30)$$

or

$$\frac{k}{\overline{\omega}_e'} \leq \frac{3.464}{\sqrt{1.33 - \overline{M}_e'' / \overline{M}_e'}}. \quad (31)$$

From the condition that the engine with  $i_c = 6$  should work in the pre-resonance zone II (see [Figure 1](#)), [relation \(12\)](#) is

valid. In view of [relation \(26\)](#), we write [inequality \(26\)](#) in the form

$$\frac{k}{\overline{\omega}_e'} > 3. \quad (32)$$

Thus, the value  $k/\overline{\omega}_e'$  should be within

$$3 < \frac{k}{\overline{\omega}_e'} \leq \frac{3.464}{\sqrt{1.33 - \overline{M}_e'' / \overline{M}_e'}}. \quad (33)$$

We assume that

$$\overline{M}_e' = \overline{M}_e''. \quad (34)$$

Taking into account [\(34\)](#), [equation \(33\)](#) takes the form

$$3 < \frac{k}{\overline{\omega}_e'} \leq 6.03. \quad (35)$$

Let us suppose that  $k/\overline{\omega}_e' = 5$ . With the indicated value  $k/\overline{\omega}_e'$  and  $i_c = 6$  value  $k/\overline{\omega}_M' = 1.67$ . In this case, when  $\overline{M}_e' = \overline{M}_e''$  from [equation \(25\)](#) we define

$$\frac{\omega_M''}{\omega_M'} = 0.64 = \frac{\overline{\omega}_e''}{\overline{\omega}_e'} \frac{i_c''}{i_c'} = \frac{8}{6} \frac{\overline{\omega}_e''}{\overline{\omega}_e'}. \quad (36)$$

Where do we find  $\overline{\omega}_e'' / \overline{\omega}_e' = 0.48$ . Thus, when replacing a 6-cylinder engine with an 8-cylinder engine while maintaining the average effective torque  $\overline{M}_e' = \overline{M}_e''$ , it is necessary to reduce the angular speed of the crankshaft in two times.

Let us consider the case of replacing a three-cylinder engine with a four-cylinder engine when using the same transmission on the same vehicle. In this case the cyclic elastic transmission efficiency can be defined as

$$\left( \eta_{tr}^{el} \right)_{cycle}^{III} = 1 - \frac{\overline{M}_e^{III}}{\left( \eta_{eng} \right)_{cycle} \left( \omega_M^{III} \right)^2 - k^2} \chi_{min}^{III} \quad (37)$$

- when working in the resonance zone I ([Figure 1](#));

$$\left( \eta_{tr}^{el} \right)_{cucle}^{IV} = 1 - \frac{\overline{M}_e^{IV}}{\left( \eta_{eng} \right)_{cycle} k^2 - \left( \omega_M^{IV} \right)^2} \chi_{min}^{IV} \quad (38)$$

- when working in the pre-resonance zone II ([Figure 1](#)).

For a 3-cylinder ICE  $\chi_{min}^{III} = 0.259$ , and for a 4-cylinder -  $\chi_{min}^{IV} = 0.092$ .

From the condition.

$$\left( \eta_{tr}^{el} \right)_{cucle}^{III} = \left( \eta_{tr}^{el} \right)_{cucle}^{IV} \quad (39)$$

Let us define

$$\frac{\omega_M^{IV}}{\omega_M^{III}} = \sqrt{\frac{k^2}{\left( \omega_M^{III} \right)^2} - \frac{\chi_{min}^{IV}}{\chi_{min}^{III}} \frac{\overline{M}_e^{IV}}{\overline{M}_e^{III}} \left[ 1 - \frac{k^2}{\left( \omega_M^{III} \right)^2} \right]}. \quad (40)$$

From the condition for obtaining real values of the root in the right-hand side of [equation \(40\)](#) we define

$$\frac{\omega_M^{III}}{k} \leq \sqrt{1 + \frac{\chi_{min}^{III}}{\chi_{min}^{IV}} \frac{\overline{M}_e^{III}}{\overline{M}_e^{IV}}}. \quad (41)$$

Taking into account [equation \(4\)](#) at  $i_c = i_c^{III} = 3$ , we obtain

$$\frac{\omega_M^{III}}{k} \leq 0.67 \sqrt{1 + \frac{\chi_{min}^{III}}{\chi_{min}^{IV}} \frac{\overline{M}_e^{III}}{\overline{M}_e^{IV}}}. \quad (42)$$

Taking into account the operation of the three-cylinder engine in the resonance zone, we obtain

$$1 < \frac{\omega_M^{\text{III}}}{k} \leq \sqrt{1 + \frac{\chi_{\min}^{\text{III}}}{\chi_{\min}^{\text{IV}}} \frac{\bar{M}_e^{\text{III}}}{\bar{M}_e^{\text{IV}}}} \quad (43)$$

or

$$0.67 < \frac{\omega_M^{\text{III}}}{k} \leq 0.67 \sqrt{1 + \frac{\chi_{\min}^{\text{III}}}{\chi_{\min}^{\text{IV}}} \frac{\bar{M}_e^{\text{III}}}{\bar{M}_e^{\text{IV}}}}. \quad (44)$$

Let us suppose that  $\bar{M}_e^{\text{III}} = \bar{M}_e^{\text{IV}}$ . In this case we get

$$0.67 < \frac{\omega_M^{\text{III}}}{k} \leq 1.307. \quad (45)$$

Let us accept  $\frac{\omega_M^{\text{III}}}{k} = 1.20$ , then  $\frac{\omega_M^{\text{IV}}}{\omega_M^{\text{III}}} = 0.765$  and

$$\frac{\bar{\omega}_e^{\text{IV}}}{\bar{\omega}_e^{\text{III}}} = \frac{\omega_M^{\text{IV}} i_c^{\text{III}}}{\omega_M^{\text{III}} i_c^{\text{IV}}} = 0.765 \frac{3}{4} = 0.574$$

The application of the proposed methods are performed as a preliminary consideration. In the future, we must take into account that the fulfillment of the condition  $\bar{M}_e = \bar{M}_e'$  and  $\bar{M}_e^{\text{III}} = \bar{M}_e^{\text{IV}}$  leads to a decrease in engine power. In the future, it is necessary to solve the specified problem under the conditions  $\bar{N}_e = \bar{N}_e'$  and  $\bar{N}_e^{\text{III}} = \bar{N}_e^{\text{IV}}$  (where  $\bar{N}_e$  is the average effective engine power). It should also be taken into account that the lower bounds of conditions (33) and (44) correspond to the resonance points of oscillations and they need to be increased taking into account the allowable dynamic coefficient.

## Conclusions

1. The proposed methods make it possible to evaluate and ensure vibration stability of vehicles with ICE.
2. Based on the previously obtained expression for the cyclic elastic efficiency of transmission located in an aggregate with ICE, a certain condition for ensuring vibration stability of a motor-transmission unit is determined. Requirements for the selection of elastic-inertial parameters of transmission are developed.
3. An analysis of the conditions for ensuring the vibration resistance of vehicles with ICE allowed to obtain the following results:
  - with the number of cylinders  $i_c$ , more than the conventional value of 3.68, the transmission must be designed so that the oscillatory processes in it flow in the pre-resonance zone;
  - when the number of cylinders  $i_c$  is less than the conventional value of 3.68, the parameters of the oscillating system "ICE-transmission" should be chosen in such a way as to ensure its operation in the after-resonance zone.
4. The introduced generalized parameters  $\chi_{\min}$  and  $\chi$  made it possible to determine the vibration stability

zone of engine assemblies for various numbers of ICE cylinders.

5. When upgrading engine assemblies by increasing the number of ICE cylinders, for example from  $i_c = 6$  to  $i_c = 8$  with the remaining parameters unchanged, it leads to a reduction in the cyclic elastic transmission efficiency, which, depending on the parameter  $\chi$ , can be from 4% to 30%. To maintain cyclic elastic efficiency at the same level, it is necessary to increase the value of  $\chi$  by 33% when changing from  $i_c = 6$  to  $i_c = 8$ . In this case, the angular velocity of the crankshaft must be reduced by about 1.75 times.
6. The transition from a 3-cylinder ICE to a 4-cylinder due to the transition through a perfect point  $i_c = 3.68$ . In this case, the vibrations of the motor-transmission unit should move from the after-resonance zone to the before-resonance zone. This can be achieved by reducing the angular velocity of the crankshaft.
7. The considered options for the application of the proposed methods are performed as an example. In the future, it must be taken into account that the fulfillment of conditions  $\bar{M}_e = \bar{M}_e'$  and  $\bar{M}_e^{\text{III}} = \bar{M}_e^{\text{IV}}$  with a decrease in  $\omega_e$  and  $\omega_e''$  in relation to  $\omega_e'$  and  $\omega_e^{\text{IV}}$ , leads to a decrease in power  $\bar{N}_e$  and  $\bar{N}_e^{\text{III}}$  in relation to  $\bar{N}_e'$  and  $\bar{N}_e^{\text{IV}}$ .
8. In the future, it is necessary to solve the indicated problem with  $\bar{N}_e = \bar{N}_e'$  and  $\bar{N}_e^{\text{III}} = \bar{N}_e^{\text{IV}}$ . It should also be taken into account that the lower boundaries of the stable zone (expressions (33) and (44)) correspond to the resonance points of the vibrations and they need to be increased taking into account the limitation of the dynamic coefficient of the vibrational process.

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## An Overall Glossary of Terms and Definitions

$A_m$  - the amplitude of the oscillations of the indicator torque of the engine [6],

$\bar{M}_i$  - average indicator engine torque;

$K_i$  - coefficient of uneven torque [6]

$\bar{\omega}_e$  - the average angular velocity of the crankshaft of ICE;

$\omega_m$  - the circular frequency of torque fluctuations of engine,

$i_c$  - the number of engine cylinders;

$k$  - the circular frequency of natural vibrations of the input shaft of the transmission,

$c_{IIp}; I_{IIp}$  - stiffness and moment of inertia of the oscillating system "ICE - transmission - wheeled vehicle - road" brought to the input shaft.

$\bar{M}_e$  - the average effective torque of the ICE;

$(\eta_{eng})_{cycle}$  - cyclic mechanical engine efficiency.

$(\eta_{tr}^{el})'_{cycle}; (\eta_{tr}^{el})''_{cycle}$  - are cyclic elastic transmission efficiencies at  $i_c = 6$  and  $i_c = 8$ .

$\chi'; \chi''$  - are the values of the generalized parameters with  $i_c = 6$  to  $i_c = 8$ .